経済・経営のための数学 a1

練習問題 2 解答例

1.

(a) $(27)^{\frac{2}{3}} = (3^2 \times 3)^{\frac{2}{3}}$

$= (3^3)^{\frac{2}{3}} = 3^2 = 9$

(b) $\left(\frac{1}{32}\right)^{\frac{2}{5}} = \left(\frac{1}{4^2 \times 2}\right)^{\frac{2}{5}}$

$= \left(\frac{1}{2^4 \times 2}\right)^{\frac{2}{5}}$

$= (2^{-5})^{\frac{2}{5}}$

$= 2^{-2} = \frac{1}{4}$

(c) $(16^{\frac{1}{4}})^{-2} = \{(4^2)^{\frac{1}{4}}\}^{-2}$

$= (4^{\frac{1}{2}})^{-2} = 4^{-1}$

$= \frac{1}{4}$

(d) $(100)^{-0.25}$

$= (10^2)^{-\frac{1}{4}} = 10^{-\frac{1}{2}} = \frac{1}{\sqrt{10}} \left(= \frac{\sqrt{10}}{10}\right)$.

(e) $(0.01)^{-0.5}$ $\frac{1}{2}$

$= (10^{-2})^{-\frac{1}{2}} = 10^{1} = \sqrt{10}$ 10

(f) $\left(\frac{4}{9}\right)^{-\frac{1}{2}} = \left\{\left(\frac{2}{3}\right)^2\right\}^{-\frac{1}{2}} = \left(\frac{2}{3}\right)^{-1}$

$= \frac{3}{2}$

(g) $6^3 \times 2^{-3} \times 3^{-2}$

$= (2\times 3)^3 \times 2^{-3} \times 3^{-2}$

$= 2^3 \times 3^3 \times 2^{-3} \times 3^{-2}$

$= 1 \times 3^1 = 3$

(h) $\left(-\frac{3}{4}\right)^{-3} \times \left(\frac{9}{8}\right)^2$

$= (-1)^{-3} \times 3^{-3} \times 4^3 \times 9^2 \times 8^{-2}$

$$= -1 \times 3^{-3} \times 3^{4} \times 2^{6} \times 2^{-6}$$

$$= -3$$

(h) $(4^{\frac{1}{3}} + 2^{\frac{1}{3}})(16^{\frac{1}{3}} - 8^{\frac{1}{3}})$

$$= (2^{\frac{2}{3}} + 2^{\frac{1}{3}})(2^{\frac{4}{3}} - 2^{1})$$

$$= 2^{\frac{2}{3}+\frac{4}{3}} - 2^{\frac{2}{3}+1} + 2^{\frac{1}{3}+\frac{4}{3}} - 2^{\frac{1}{3}+1}$$

$$= 2^{2} - 2^{\frac{5}{3}} + 2^{\frac{5}{3}} - 2^{\frac{5}{3}}$$

$$= 2^{2} - 2^{\frac{5}{3}}$$

(j) $(6\times4)^{\frac{1}{2}} + \left(\frac{3}{2}\right)^{\frac{1}{2}} - \left(\frac{2}{75}\right)^{-\frac{1}{2}}$

$75 = 3 \times 25$

$$= 2(2\times3)^{\frac{1}{2}} + \left(\frac{3}{2}\right)^{\frac{1}{2}} - \left(\frac{2}{3} \times \frac{1}{5^{2}}\right)^{-\frac{1}{2}}$$

$$= 2(2\times3)^{\frac{1}{2}} + \left(\frac{3}{2}\right)^{\frac{1}{2}} - \left(\frac{3}{2}\right)^{\frac{1}{2}} 5$$

$$= 2 \times 6^{\frac{1}{2}} + \left(\frac{3}{2}\right)^{\frac{1}{2}} (1 - 5)$$

$$= 2 \times 6^{\frac{1}{2}} - \left(\frac{3}{2}\right)^{\frac{1}{2}} \times 2^2$$

$$= 2 \times 2^{\frac{1}{2}} \times 3^{\frac{1}{2}} - 3^{\frac{1}{2}} \times 2^{-\frac{1}{2}} \times 2^2$$

$$= 2^{\frac{3}{2}} \times 3^{\frac{1}{2}} - 3^{\frac{1}{2}} \times 2^{\frac{3}{2}}$$

$$= 0.$$

(k) $$\left(\frac{125}{8}\right)^{-\frac{1}{2}} \times \frac{5}{4}$$

$$= \left(\frac{5^3}{2^3}\right)^{-\frac{1}{2}} \times \frac{5}{8}$$

$$= 5^{-\frac{3}{2}} \times 2^{\frac{3}{2}} \times 5 \times 2^{-3}$$

$$= 5^{-\frac{1}{2}} \times 2^{-\frac{3}{2}}$$

$$\left(= \frac{1}{2(5 \times 2)^{\frac{1}{2}}} = \frac{\sqrt{10}}{20} \right)$$

(l) $(a^{\frac{1}{2}}b^{-\frac{2}{3}})^{-2} \times (ab^{-1})^{\frac{1}{3}}$

$= a^{-1} \times b^{\frac{4}{3}} \times a^{\frac{1}{3}} \times b^{-\frac{1}{3}}$

$= a^{-\frac{2}{3}} b$

(m) $a^{\frac{2}{3}} b^{\frac{1}{2}} (a^{\frac{4}{3}} b^{-\frac{1}{2}} + a^{-\frac{2}{3}} b^{\frac{5}{2}})$

$= a^{\frac{2}{3}+\frac{4}{3}} b^{\frac{1}{2}-\frac{1}{2}} + a^{\frac{2}{3}-\frac{2}{3}} b^{\frac{1}{2}+\frac{5}{2}}$

$= a^2 b^0 + a^0 b^3$

$= a^2 + b^3$

2.

(a) $\log_8 64^{\frac{1}{3}} = \frac{1}{3} \log_8 8^2$

$= \frac{2}{3}$

(b) $\log_2 \frac{1}{32}$

$= \log_2 (4\times 8)^{-1}$

$= -\log_2 2^5$

$= -5$

(c) $\log_{10} (10)^{-3}$

$= -3$

(d) $\log_{25} 5^{\frac{1}{2}}$

$= \log_{25} \{(25)^{\frac{1}{2}}\}^{\frac{1}{2}}$

$= \frac{1}{4}$.

(e) $\log_{\frac{1}{2}} 8 = 3 \log_{\frac{1}{2}} 2$

$= 3 \log_{\frac{1}{2}} (\frac{1}{2})^{-1} = -3$

(f) $\log_3 \frac{5}{3} + \log_3 \frac{27}{5}$

$= \log_3 5 - \log_3 3$

$+ \log_3 27 - \log_3 5$

$= 3 - 1 = 2$.

(g) $\log_3 12 + \log_3 36 - \log_3 16$

$= \log_3 4 + \log_3 3$

$2 \log_3 6 - 4 \log_3 2$

$= 2 \log_3 2 + 1$

$+ 2 (\log_3 2 + \log_3 3)$

$- 4 \log_3 2$

$= 3$

(h) $2\log_{10} 5^{\frac{1}{2}} + \frac{1}{2}\log_{10} 2^2$

$= \log_{10} 5 + \log_{10} 2$

$= \log_{10} 5 + \log_{10} 10/5$

$= 1$

(i) $\frac{1}{2}\log_{10} 3 + 3\log_{10} 2^{\frac{1}{2}} - \log_{10} 6^{\frac{1}{2}}$

$= \underbrace{\frac{1}{2}\log_{10} 3 + \frac{3}{2}\log_{10} 2}_{A} - \log_{10}(2\times 3)^{\frac{1}{2}}$

$= A - \frac{1}{2}\log_{10} 2 - \frac{1}{2}\log_{10} 3$

$= \log_{10} 2$

(j) $\log_3 (3^2 \times 2^3)^{\frac{1}{2}} - \log_3 2^2 - \log_3 \frac{3^3}{6^{\frac{1}{2}}}$

$$= \log_3 3 + \log_3 2^{\frac{3}{2}} - \log_3 2^2$$

$$- \log_3 3^3 + \log_3 6^{\frac{1}{2}}$$

$$= 1 + 3/2 \log_3 2 - 2 \log_3 2$$

$$- 3 + \frac{1}{2} \log_3 2 + \frac{1}{2} \log_3 3$$

$$= -2 + \frac{1}{2} = -\frac{3}{2} \quad //$$

3.

(a) $4^{x-1} = 2^{7-x}$

$$2^{2x-2} = 2^{7-x}$$

$\therefore$ $2x - 2 = 7 - x$

$3x = 9$

$x = 3$

//

(b) $(2^x - 16)(2^{-x} - 4) = 0$

$$\begin{cases} 2^x = 16 & ① \\ \text{or} \\ 2^{-x} = 4 & ② \end{cases}$$

① のとき $2^x = 2^4$ $\quad x = 4$

② のとき.

$2^{-x} = 2^2$ $\quad x = -2$

$\therefore$ $x = -2,\ 4$

(c) $3^{2x} - 25 \times 3^x - 54 = 0$

$3^x = t$ とおく.

$t^2 - 25t - 54 = 0$

$(54 = 6 \times 9 = 2 \times 3 \times 3 \times 3)$

$(t - 27)(t + 2) = 0$ より

$3^x = 27$ or $3^x = -2$

→ あり得ない.

$\therefore \quad 3^x = 27$

$3^x = 3^3 \quad \therefore \quad x = 3$

(d) $\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-2x+3}$

$x = -2x + 3$

$x = 1$

(e) $\log_x 243 = \frac{5}{3}$

$x^{\frac{5}{3}} = 243 = 3 \times 81 = 3^5$

$\therefore \quad x^5 = (3^5)^3$ (両辺を3乗)

$x^5 = (3^3)^5$

$\therefore \quad x = 3^3 = 27$ (両辺を$\frac{1}{5}$乗)

別解

$$\log_x 243 = \frac{5}{3}$$

$$\Leftrightarrow \log_x 3^5 = \frac{5}{3}$$

$$\Leftrightarrow 5\log_x 3 = \frac{5}{3}$$

$$\Leftrightarrow \log_x 3 = \frac{1}{3}$$

$$\therefore x^{\frac{1}{3}} = 3 \quad \text{両辺を } (\ )^3$$

$$x = 27$$

(f) $\log_4 0.0625 = x$

$$\log_4 \frac{625}{10000} = \log_4 \frac{5\times 125}{10^4}$$

$$= \log_4 \frac{5^4}{10^4}$$

$$= \log_4 \left(\frac{1}{2}\right)^4 \quad \text{底を } 4$$

$$4\log_4 \frac{1}{2} = 4\log_4 4^{-\frac{1}{2}} = -2 = x$$

(g) $\log_3(2x-1) = 2\log_3 x$

$= \log_3 x^2$

$\therefore \quad x^2 = 2x - 1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0 \qquad \therefore x = 1$

(h) $\log_{10} 10(x-1) = \log_{10}(x^2 - 5x + 4)$

$10x - 10 = x^2 - 5x + 4$

$0 = x^2 - 15x + 14$

$0 = (x-14)(x-1)$

* $\log_a x > 0$ 与 $x \neq 1$.

$\therefore \quad x = 14$

(i) $(\log_{10} x)^2 - 2\log_{10} x = 3$

$\log_{10} x = t$ とおくと

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0$

$t = -1,\ 3$

$\therefore \log_{10} x = -1 \cdots ①$

or

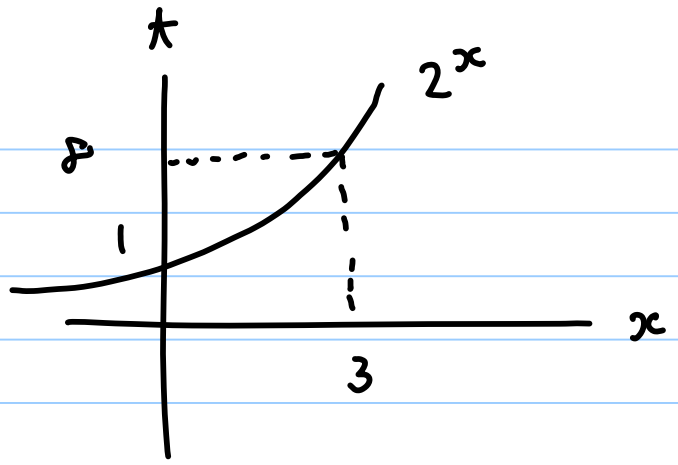
$\log_{10} x = 3 \cdots ②$

$\begin{cases} ① \text{のとき} & \dfrac{1}{10} = x \\ ② \text{のとき} & x = 10^3 = 1000 \end{cases}$

4. $t = 2^x$ とおくと,

$y = 4t - t^2 = -t(t-4)$

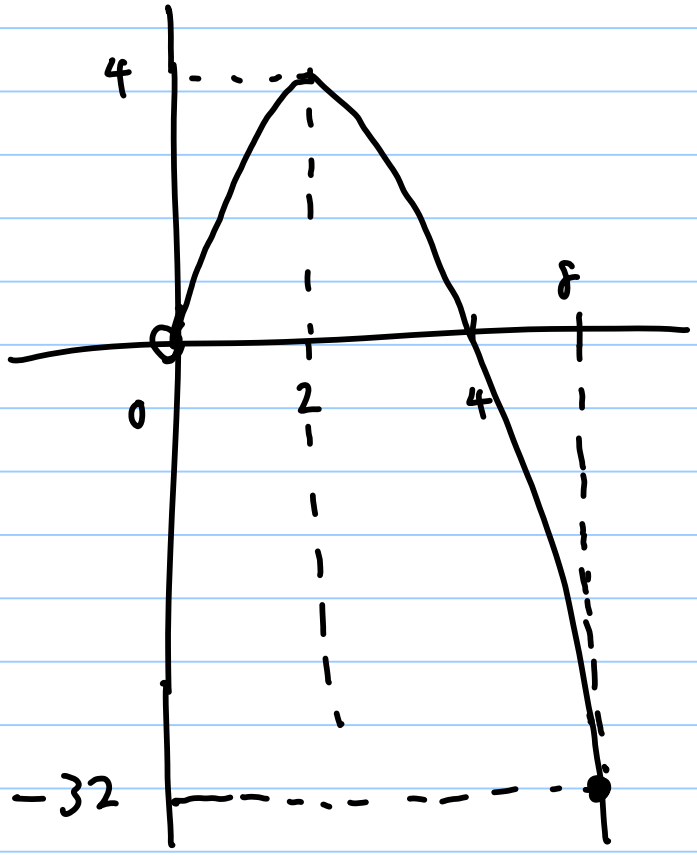
であることと, t の値域は $x \leq 3$ より



$0 < t \leq 8$ が分かる

よって, $y = -t(t-4)$ $(0 < t \leq 8)$

の max and min を考えればよい.



$y = -\{t^2 - 4t\}$

$= -\{(t-2)^2 - 4\}$

$= -(t-2)^2 + 4$

∴ min -32 $(x=3$ のとき$)$

max 4 $(x=1$ のとき$)$

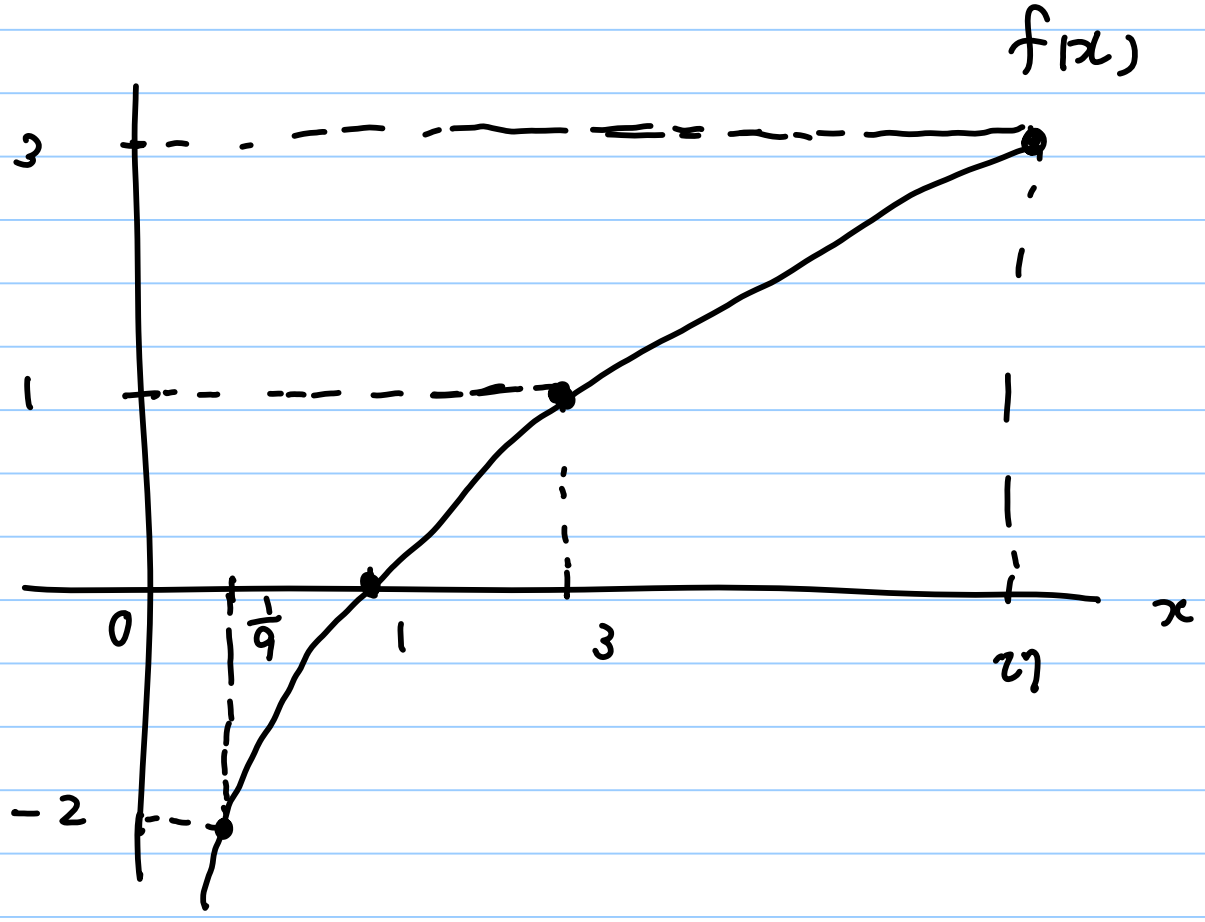
5. $f(x) = \log_3 x$

$f(\frac{1}{9}) = \log_3 3^{-2} = -2$

$f(1) = \log_3 1 = 0$

$f(3) = \log_3 3 = 1$

$f(27) = \log_3 3^3 = 3$



6. $y = \log_4 (x-1) + 2$

$y - 2 = \log_4 (x-1)$

$4^{y-2} = x - 1$

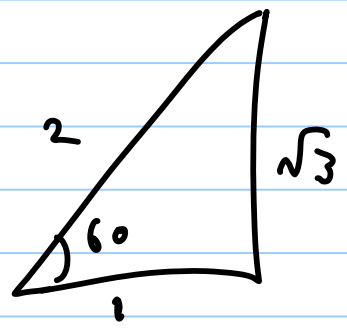
$x = 4^{y-2} + 1 = f^{-1}(y)$

$\therefore y = f^{-1}(x) = 4^{x-2} + 1$ //

7.

(a) $\frac{\pi}{6} = 30°$

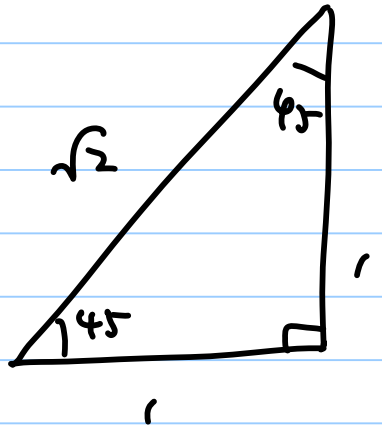
$\begin{cases} \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} \\ \cos \frac{\pi}{6} = \frac{1}{2} \\ \tan \frac{\pi}{6} = \sqrt{3} \end{cases}$



(1A)

(b) $\theta = \frac{\pi}{4} = 45°$

$$\begin{cases} \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \\ \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \\ \tan\frac{\pi}{4} = 1 \end{cases}$$

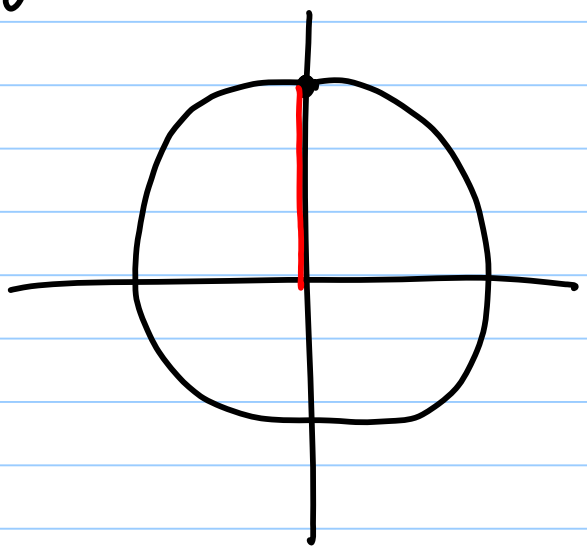


(c) $\theta = \frac{\pi}{2} = 90°$

$\sin\frac{\pi}{2} = 1$

$\cos\frac{\pi}{2} = 0$

$\tan\frac{\pi}{2}$: なし



斜辺 = 高さ

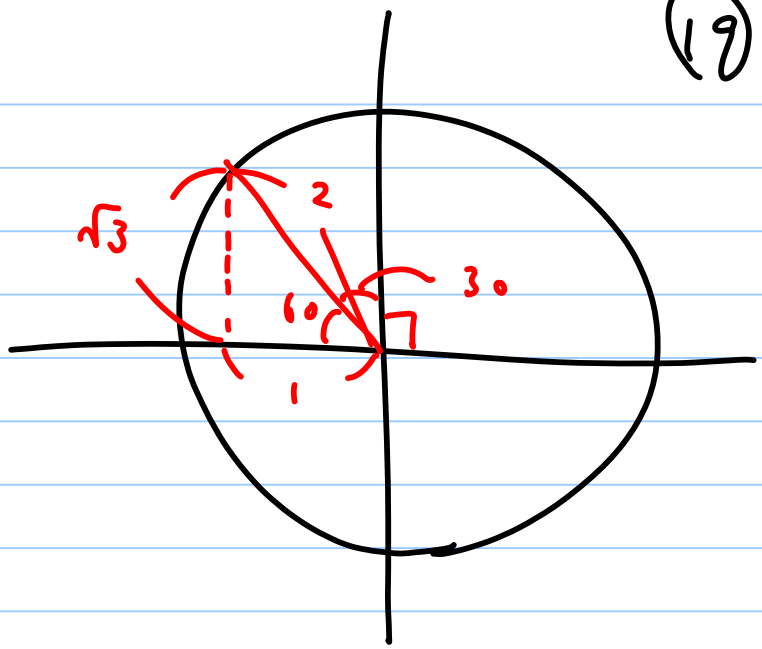
横 = 0

(d) $\theta = \frac{2}{3}\pi = 120°$

$\sin \frac{2}{3}\pi = \frac{\sqrt{3}}{2}$

$\cos \frac{2}{3}\pi = -\frac{1}{2}$

$\tan \frac{2}{3}\pi = -\sqrt{3}$



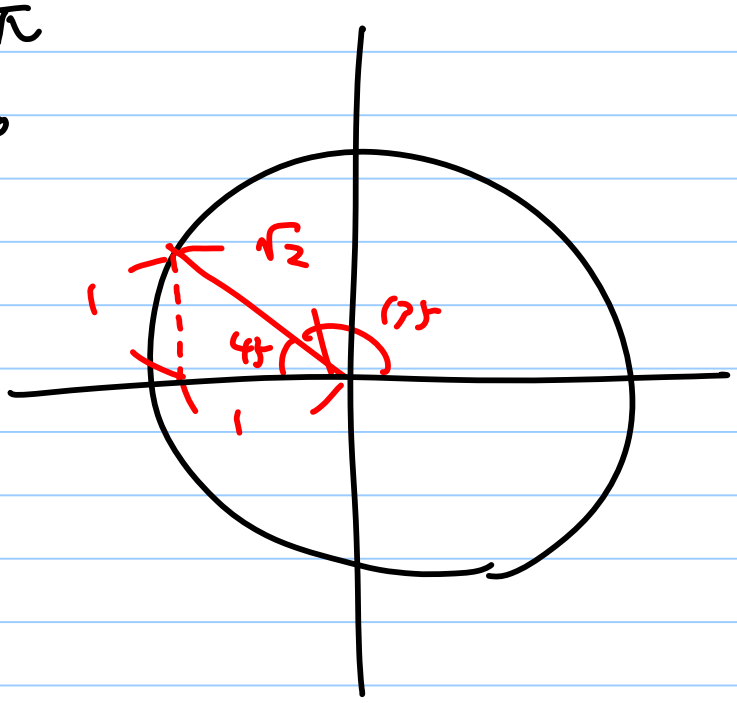
(e) $\theta = \frac{3}{4}\pi$

$= 135°$

$\sin \frac{3}{4}\pi = \frac{1}{\sqrt{2}}$

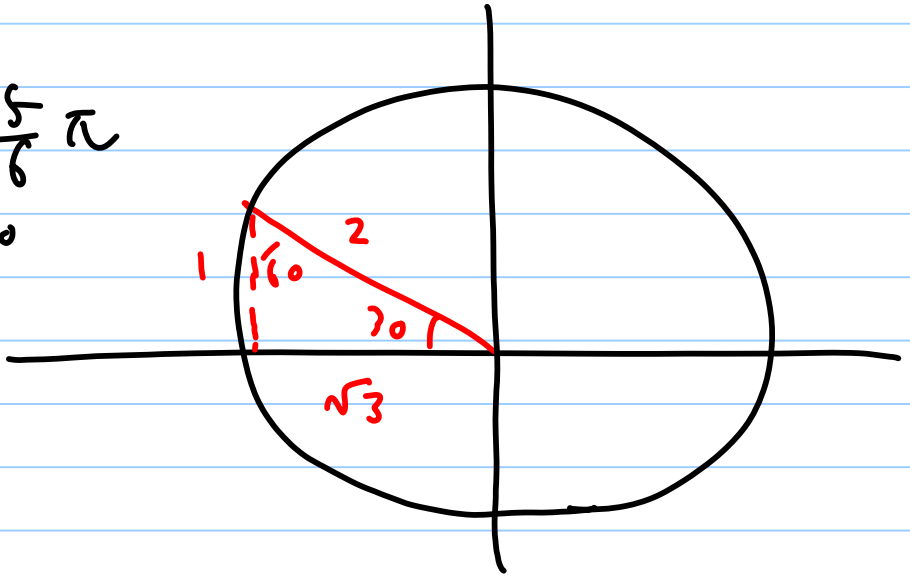
$\cos \frac{3}{4}\pi = -\frac{1}{\sqrt{2}}$

$\tan \frac{3}{4}\pi = -1$



(f) $\theta = \frac{5}{6}\pi$

$= 150°$



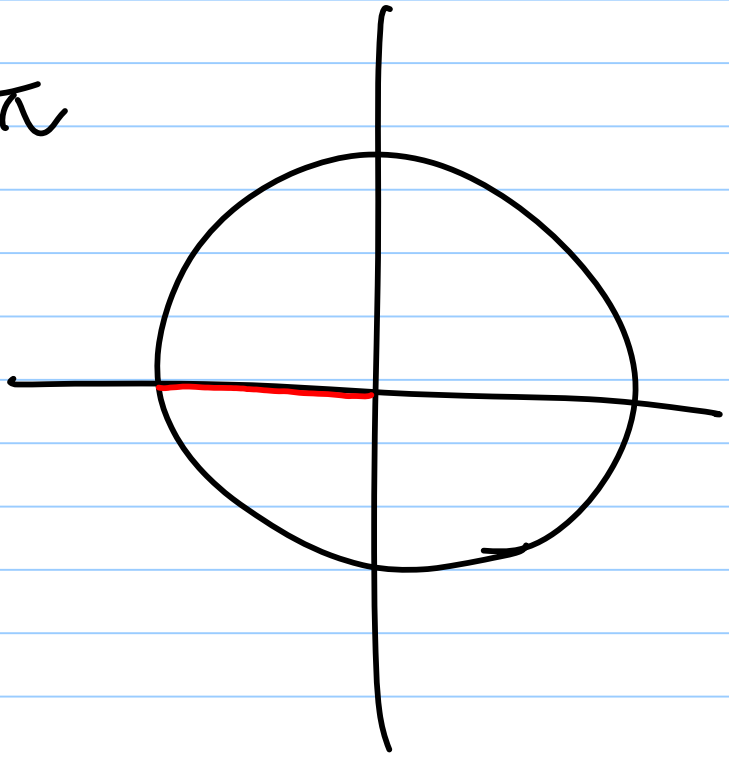
(20)

$\sin \frac{5}{6}\pi = \frac{1}{2}$

$\cos \frac{5}{6}\pi = -\frac{\sqrt{3}}{2}$

$\tan \frac{5}{6}\pi = -\frac{1}{\sqrt{3}}$

(g) $\theta = \pi$



$\sin \pi = 0$

$\cos \pi = -1$

$\tan \pi = 0$

斜辺 = よこ

(ただし⊖)

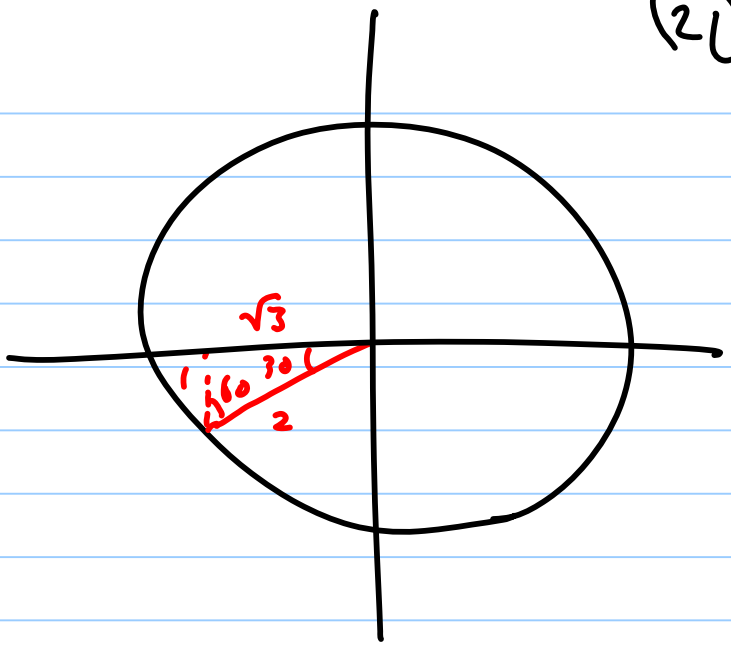
高さ 0

(h) $\theta = \frac{11}{6}\pi = 210°$

$$\sin \frac{7}{6}\pi = -\frac{1}{2}$$

$$\cos \frac{7}{6}\pi = -\frac{\sqrt{3}}{2}$$

$$\tan \frac{7}{6}\pi = \frac{1}{\sqrt{3}}$$

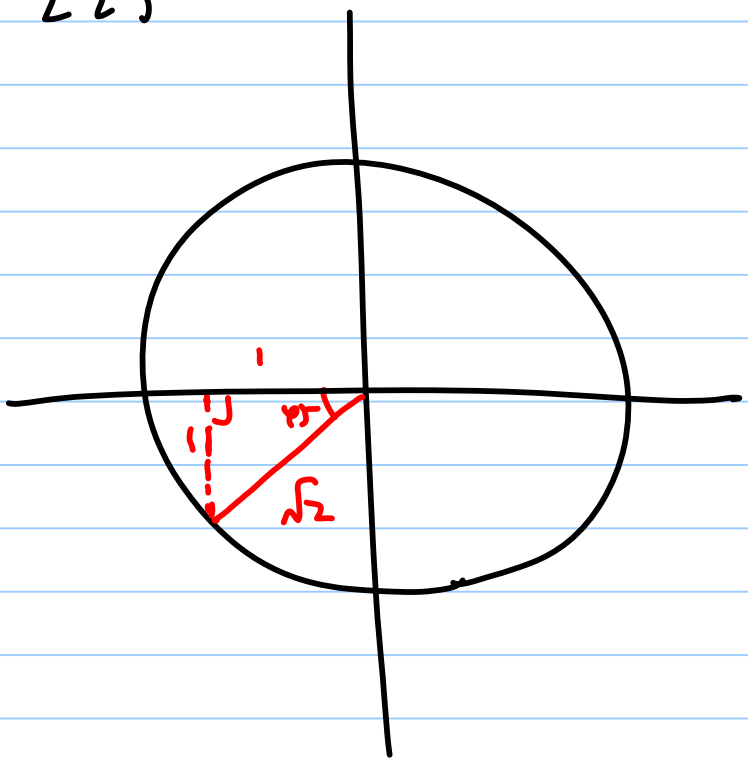


(ii) $\theta = \frac{5}{4}\pi = 225°$

$$\sin \frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$$

$$\cos \frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$$

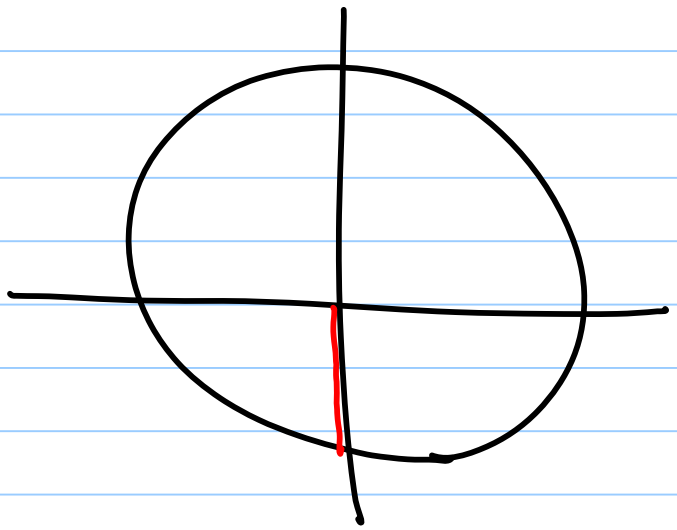
$$\tan \frac{5}{4}\pi = 1$$



(iii) $\theta = \frac{3}{2}\pi$

$$\sin \frac{3}{2}\pi = 1$$

$$\cos \frac{3}{2}\pi = 0$$



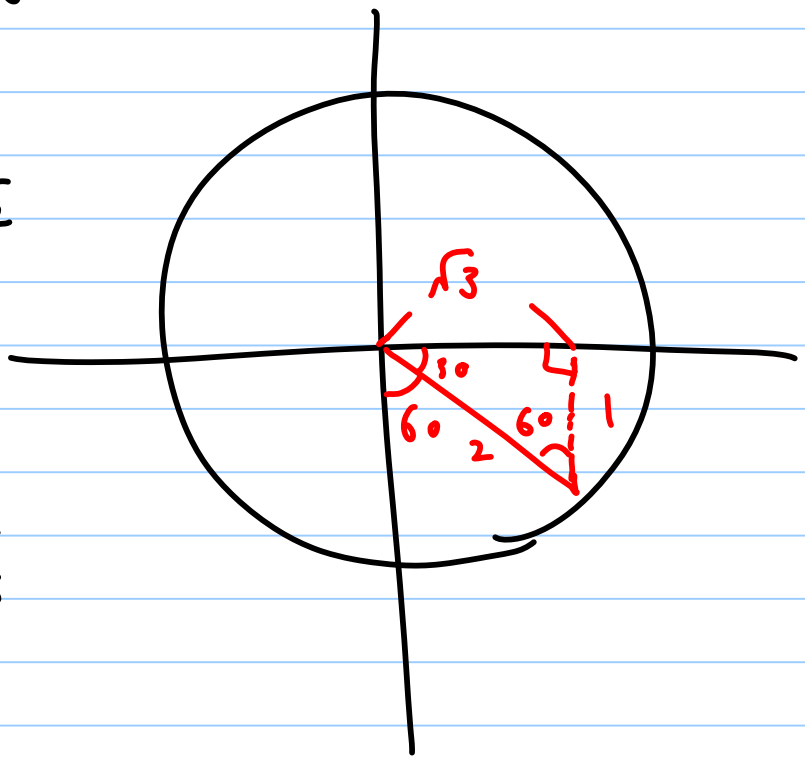
tan $\frac{3}{2}\pi$ 는 $\pm\infty$

(h) $\theta = \frac{11}{6}\pi = 330°$

$\sin\frac{11}{6}\pi = -\frac{1}{2}$

$\cos\frac{11}{6}\pi = \frac{\sqrt{3}}{2}$

$\tan\frac{11}{6}\pi = -\frac{1}{\sqrt{3}}$



(l) $\theta = \frac{11}{4}\pi$

$\frac{11}{4}\pi = \frac{3}{4}\pi + 2\pi$ 사분

$\sin\frac{11}{4}\pi = \frac{1}{\sqrt{2}}$

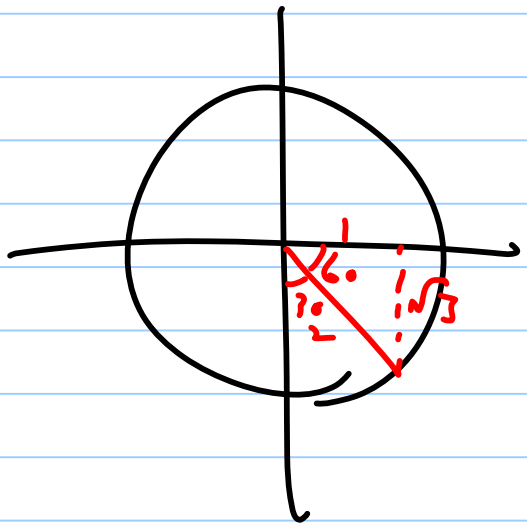
$\cos\frac{11}{4}\pi = -\frac{1}{\sqrt{2}}$

$\tan\frac{11}{4}\pi = -1$

(m) $\theta = -\frac{1}{3}\pi$

$-\frac{1}{3}\pi = \frac{5}{3}\pi - 2\pi$

IV



$\sin -\frac{1}{3}\pi = -\frac{\sqrt{3}}{2}$

$\cos -\frac{1}{3}\pi = \frac{1}{2}$

$\tan -\frac{1}{3}\pi = -\sqrt{3}$

(n) $\theta = -\frac{5}{4}\pi$

$-\frac{5}{4}\pi = \frac{3}{4}\pi - 2\pi$ IV

$\sin\left(-\frac{5}{4}\pi\right) = \frac{1}{\sqrt{2}}$

$\cos\left(-\frac{5}{4}\pi\right) = -\frac{1}{\sqrt{2}}$

$\tan\left(-\frac{5}{4}\pi\right) = -1.$